

**THE THEORETICAL INTEREST IN ELASTIC  
SCATTERING WITH POLARIZED BEAMS**

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**ABSTRACT:** We review briefly the interesting physics questions that can be addressed by studying the high energy elastic scattering of polarized proton and antiproton beams at both small and large momentum transfer. An extremely rich and informative program seems possible in the near future.

Invited talk, Symposium on Future Polarization Physics at Fermilab.

June 1988

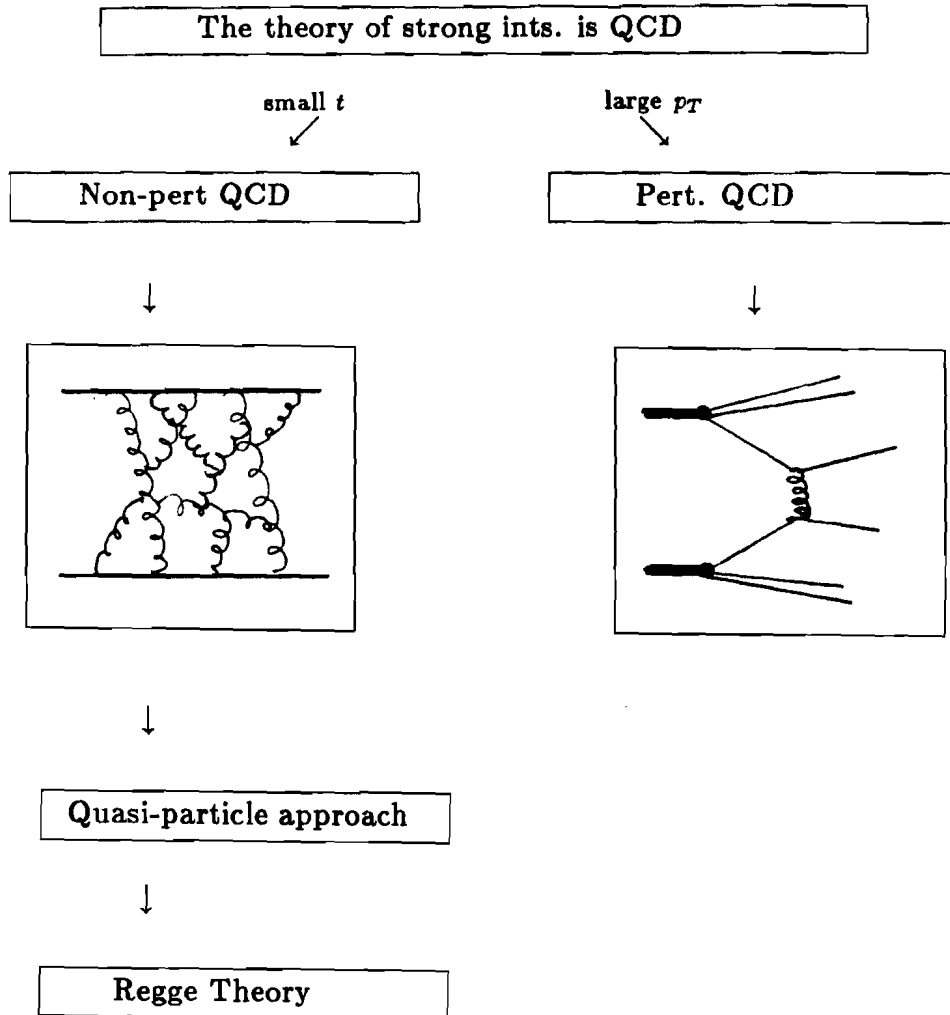
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### Introduction

We assume that strong interactions are governed by QCD. In that case the scenario for discussing small and large momentum scattering is schematically as shown below:

Assumption:



For large  $p_T$  the partonic scattering process is reasonably straightforward to calculate, and the partonic wave functions of the hadron are supposed to be obtained from a mixture of theory and information from experimental measurements like deep inelastic lepton-hadron scattering.

For small  $t$  there is no hope at present for detailed dynamical calculation and the sensible approach is via "quasiparticles" which hopefully mirror the essential dynamical degrees of freedom which are important in this kinematical region. The correct approach is almost certainly that of Regge pole theory. Interesting progress has been made recently in demonstrating how QCD gives rise to Regge pole behavior.<sup>1</sup>

#### The region of small $t$

The classic Regge pole amplitude for an arbitrary  $2 \rightarrow 2$  helicity amplitude is<sup>2</sup>

$$H_{cd;ab}(s, t) = \gamma_{ca}(t)\gamma_{db}(t) \left[ \frac{1 + \tau e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} \right] \left( \frac{s}{s_0} \right)^{\alpha(t)}. \quad (1)$$

The labels  $a, b, c, d$  refer to the helicities of the particles in the reaction  $A + B \rightarrow C + D$ . The elegant expression (1) is one of the greatest achievements in strong interaction physics.

The positive aspects of this beautiful and concise result are that:

- a) The particle mass spectrum tells us about the trajectory function  $\alpha(t)$ .
- b) The amplitude has a non-zero phase; a great improvement over lowest order calculations and of vital importance for spin parameters.

The negative aspects are:

- a) We do not know how to calculate (from QCD) the  $t$ -dependence of the residue function  $\gamma_{ca}(t)$ .
- b) More complicated exchanges of several Regge poles are sometimes important. These are hard to calculate precisely.

A mixed blessing is the remarkable factorization property:<sup>3</sup>  $H_{cd;ab}$  is made up of a product, each factor referring to one pair of particles only viz:  $(ca) \times (db)$ .

Generally speaking Regge poles and cuts give a reasonable account of medium energy phenomenology,<sup>4</sup> though there are certain puzzling results that seem difficult to accommodate. However many crucial aspects of (1) have never been tested and certain folkloric properties have been used over and over again without any real theoretical basis. It is not clear, for example, whether we understand the Regge pole corresponding the most basic elementary particle, the pion. The new range of polarization-type experiments could help to clarify some of these issues. We mention just a few:

The pomeron  $IP$  and the odderon  $O$

Can the  $IP$  flip helicity? What, if any, is the role of the odderon? (Recall that the  $O$  is the odd-signature analogue of the  $IP$ , whose existence is signalled by the lack of equality between  $pp$  and  $\bar{p}p$  differential cross sections at ISR energies<sup>5</sup> and by the unexpectedly large real to imaginary forward ratio in  $\bar{p}p$  at  $\sqrt{s} = 546$  GeV.<sup>6</sup>)

The rate of growth with energy is the same for  $IP$  and  $O$  but they are roughly  $90^\circ$  out of phase. The latter is perfect for producing a large analyzing power (or polarization), since

$$A \approx \text{Im} [\phi_5^*(\phi_1 + \phi_3)]. \quad (2)$$

But, does the  $IP$  and/or the  $O$  couple to the helicity-flip amplitude  $\phi_5$ ? We don't know. If not, one expects

$$A \propto \frac{1}{\sqrt{s}} \quad \text{at fixed } t. \quad (3)$$

The data (Fig. 1) may or may not support this.

In the  $IP - O$  scenario one would have

$$A \sim \text{constant} \quad \text{at fixed } t \quad (4)$$

but the numerical value would presumably be very small. In addition, for a polarized  $\bar{p}$  beam, one would expect

$$A_{\bar{p}p} \sim -A_{pp}. \quad (5)$$

Charge exchange  $\bar{p}p \rightarrow \bar{n}n$ 

We have

$$\left( A \frac{d\sigma}{dt} \right)_{\bar{p}p}^{CEX} \propto \text{Im} \{ \phi_5^* [(\phi_1 + \phi_3) + (\phi_2 - \phi_4)] \}_{I=1}. \quad (6)$$

All exchanges must have  $I = 1$ . We expect  $\rho$  and  $a_2$  (old  $A_2$ ) to dominate  $\phi_5$ . There is then no obvious  $I = 1$  contribution to  $(\phi_1 + \phi_3)$  which is out of phase with  $\phi_5$ , so one usually assumes dominance of the  $\pi$  and  $b_1$  (old  $B$ ) contribution to  $(\phi_2 - \phi_4)$ . But

$$\phi_2^{\pi, b_1} = \phi_4^{\pi, b_1} \quad (7)$$

so, naively, we expect the  $CEX$  analyzing power to be zero.

But we do *not* really understand the reggeized  $\pi$ . For example we know that  $\Delta\sigma_T \propto \text{Im}\phi_2(t=0)$  is not zero, at least at medium energies, yet the naive pion contribution to  $\phi_2$  vanishes at  $t=0$ .

So the question is an open one and (6) will help us to understand the nature of the pion Regge pole.

Spin correlations:  $A_{NN}$ 

In general,

$$A_{NN} \frac{d\sigma}{dt} \propto \text{Re} \left\{ (\phi_1 + \phi_3)(\phi_4^* - \phi_2^*) + (\phi_3 - \phi_1)(\phi_4^* + \phi_2^*) - |\phi_5|^2 \right\}. \quad (8)$$

At small  $t$  we can neglect  $\phi_5$  and  $\phi_4$ , and believe that  $\phi_1 \approx \phi_3$ , so that

$$A_{NN} \frac{d\sigma}{dt} \stackrel{\text{small } t}{\sim} \text{Re} \{ (\phi_1 + \phi_3) \phi_2^* \}. \quad (9)$$

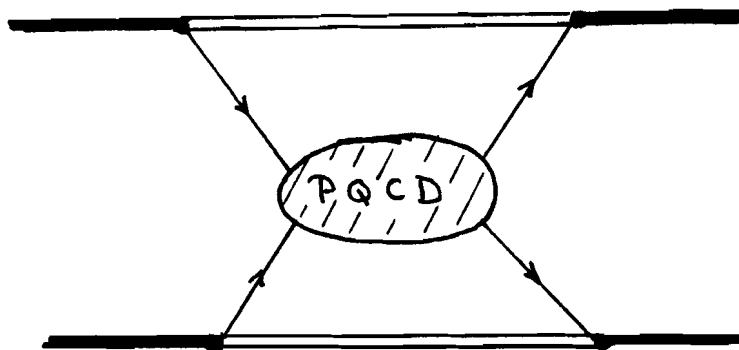
The important contributions to  $\phi_1 + \phi_3$  are  $\mathbb{P}$ , possibly  $\mathbb{O}, \omega, a_2$ . As mentioned above, we don't really understand  $\phi_2(t)$  for small  $t$ .

The energy variation, and the comparison of  $\bar{p}p$  with  $pp$  will yield valuable information on this question. This will be complementary to what is learned in (6) because here we study the real part of a product of amplitudes.

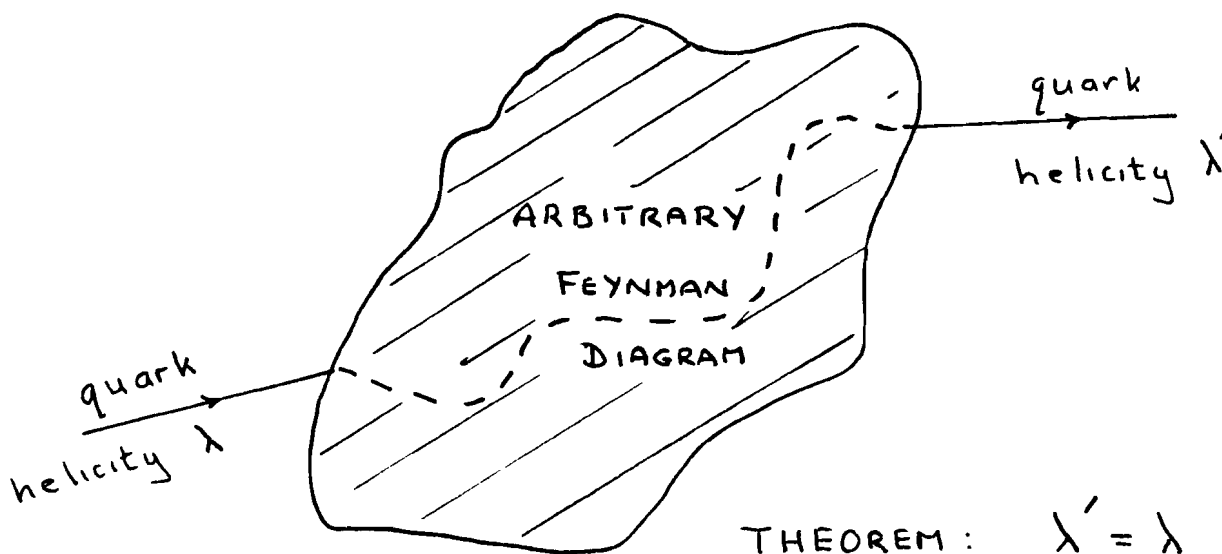
The above is a very brief summary of what is interesting. Many other aspects are discussed in the review paper of Bourrely, Leader and Soffer.<sup>4</sup> Detailed critical tests for Regge poles and a discussion of  $IP$  helicity-flip are given in Ref. 7.

The region of large  $p_T$

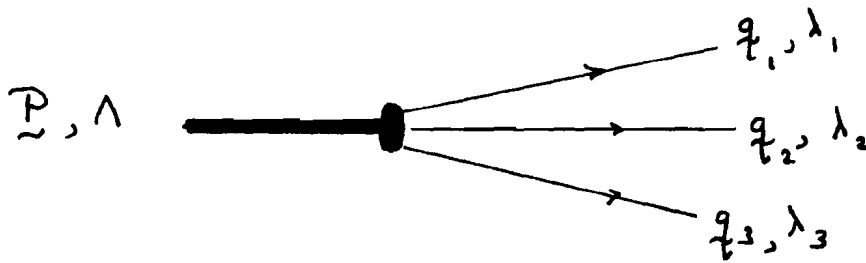
We assume the canonical picture of large  $p_T$  exclusive reactions shown below:



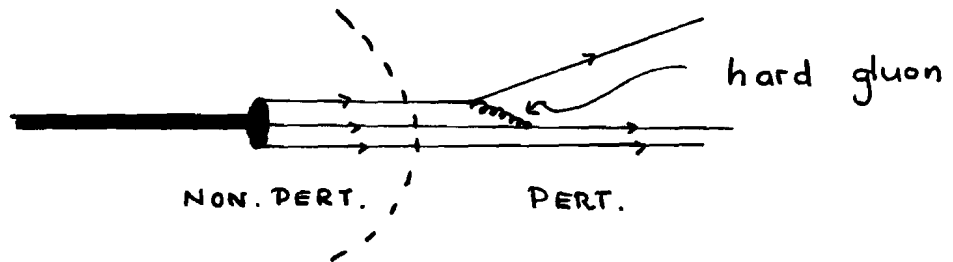
Regarding the perturbative calculation of parton-parton scattering we note the powerful theorem that holds if the quarks are taken to be massless: A quark with helicity  $\lambda$  entering an *arbitrary* Feynman diagram will emerge with its helicity unaltered.



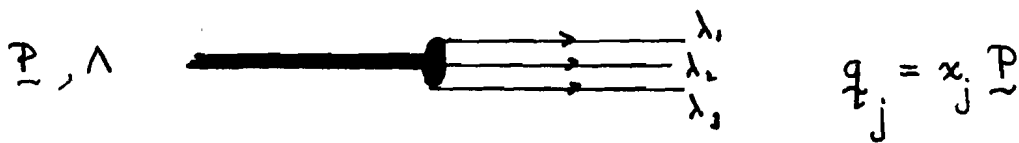
To link up with the hadronic world we require the wave function



For large  $p_T$  it is dominated by



and the scattering is computed perturbatively. So, roughly, we need only the wave function for the parallel configuration



Since the parallel quarks can have no orbital angular momentum along  $OZ$ , conservation of  $J_z$  implies

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3. \quad (10)$$

#### The polarization at large $p_T$

A dramatic consequence of this and the helicity conservation theorem is the Brodsky-

Lepage result:<sup>8</sup> In  $A + B \rightarrow C + D + E \dots$  one must have

$$\lambda_A + \lambda_B = \lambda_C + \lambda_D + \lambda_E + \dots \quad (11)$$

*i.e.*, total helicity is conserved.

An immediate consequence is that the polarization in nucleon-nucleon scattering should be zero. The polarization is proportional to  $\phi_5$  which corresponds to the transition

$$\left| \frac{1}{2}, \frac{1}{2} \right\rangle \rightarrow \left| \frac{1}{2}, -\frac{1}{2} \right\rangle.$$

The initial helicity = 1, final helicity = 0 so that  $\phi_5 = 0$ .

As Fig. 2 shows, there is no sign of the polarization vanishing as  $p_T$  increases.

To what extent can we escape the above conclusions? They relied heavily on  $m = 0$  for the quarks. What can we expect for helicity-flip amplitudes if  $m \neq 0$ ?

I believe that a quite reliable estimate is to take

$$H_{cd;ab} \approx H_{NON}^{FLIP} \cdot \left( \frac{m}{\hat{E}} \right)^N (\sin \theta/2)^{|\lambda-\mu|} (\cos \theta/2)^{|\lambda+\mu|} \quad (12)$$

where  $\lambda = a - b$ ,  $\mu = c - d$  and  $N$  is the total number of helicity-flips,  $N = |(a+b) - (c+d)|$ .  $\hat{E}$  is the energy of the quark in the parton-parton c.m.

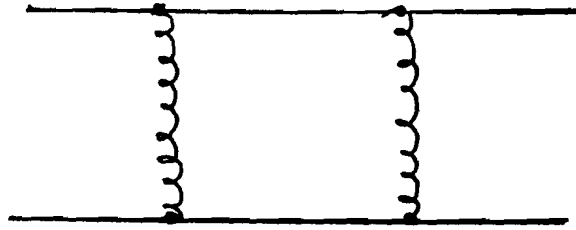
The key question is how big  $m/\hat{E}$  is. It would be incorrect, I believe, to take a fixed  $m$  for the quark and then maximize  $m/\hat{E}$  by concentrating on slow moving quarks. In the simple parton model where  $q = xP$  one should use  $m = xm_N$ ,  $\hat{E} = x\sqrt{s}/2$  so that

$$\frac{m}{\hat{E}} \sim \frac{2m_N}{\sqrt{s}} \ll 1 \quad \text{at high energies.} \quad (13)$$

Let us try now to estimate the size of the polarization. The single helicity flip implies one factor  $2m_N/\sqrt{s}$ . But our lowest order terms are real and we need a phase. So we have



to go to 4th order diagrams like



to pick up an imaginary part. This costs an extra factor of  $\alpha_S$ , so we might guess

$$P \sim \alpha_S \left( \frac{2m_N}{\sqrt{s}} \right) \sin \theta/2 \cos \theta/2. \quad (14)$$

However, as  $t \rightarrow 0$ , it is almost certain that the phases of all the  $\phi_i$  become equal.<sup>9</sup>

Thus there will be an extra factor of  $\sin \theta$ . Finally then we estimate

$$P \sim \alpha_S \left( \frac{2m_N}{\sqrt{s}} \right) \sin \theta \sin \theta/2 \cos \theta/2 f(\theta) \quad (15)$$

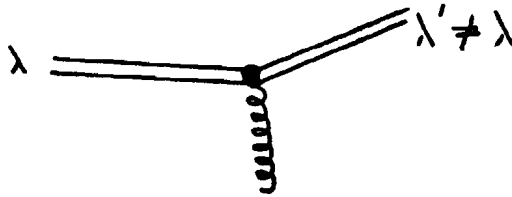
where  $f(\theta)$  reflects the detailed dynamics and  $|f(\theta)| \lesssim 1$ .

At high energies this is already a very small number. But to get the hadronic polarization we must still multiply by the polarization of the quark in the hadron, so the result will be even smaller.

Note an amusing coincidence. If there is no odderon contribution then for fixed small  $t$ ,  $P \propto 1/\sqrt{s}$ . For fixed large  $p_T$  we have also  $P \propto 1/\sqrt{s}$ .

Since the measured  $P$  shows no sign of becoming zero at large  $p_T$  one should look for higher twist effects which might still be important at Argonne and BNL energies. One way to model these effects is via diquarks.<sup>10</sup> We require the spin-one variety to be useful

in producing polarization. Indeed the gluon-diquark coupling *does* permit helicity-flip



so there is no Brodsky-Lepage rule. Thus we can get a large  $\phi_5$ . But, at present, we have no idea of the phase so cannot estimate the polarization. This should soon be calculated.

There is an interesting prediction due to Pire and Ralston.<sup>11</sup> Based upon what they call the chromo-Coulombic phase, they suggest that  $P(s, \theta)$  should oscillate with energy at fixed  $\theta$ . I do not know what the frequency of the oscillations is supposed to be theoretically, but it is suggested that it is the same as is seen<sup>12</sup> in the oscillations of the fixed-angle differential cross-sections about the simple counting-rule power behavior  $1/s^n$ .

#### Correlation parameters at large $p_T$

Consider first  $A_{NN}$ . Because of the Pauli principle  $A_{NN}^{PP}$  will not be zero even if all the helicity-flip amplitudes vanish.<sup>13</sup> For example at  $\theta = 90^\circ$ ,<sup>14</sup>

$$\begin{aligned}
 A_{NN}^{PP}(\pi/2) &= \frac{1}{1 + \frac{1}{2} |\phi_1^{PP}(\pi/2) / \phi_3^{PP}(\pi/2)|^2} \\
 &= \frac{1}{1 + 2 \left| \frac{\phi_1(\pi/2)}{\phi_3(\pi/2) - \phi_4(\pi/2)} \right|^2}
 \end{aligned} \tag{16}$$

where in the second form the amplitudes are unsymmetrized. (A reasonable guess is  $\phi_3(\pi/2) \sim \frac{1}{2}\phi_1(\pi/2)$ . In some models  $\phi_4 = 0$ . This would yield  $A_{NN} \sim 1/9$ . In others  $\phi_4(\pi/2) = -\phi_3(\pi/2)$  yielding  $A_{NN} \sim 1/3$ .)

On the contrary, for  $\bar{p}p$ , if the helicity-flip amplitudes vanish we will have

$$A_{NN}^{\bar{P}P}(\pi/2) = 0. \tag{17}$$

It will be interesting to compare these.

Note incidentally that (16) suggests that  $A_{NN}^{pp}(\pi/2)$  could become independent of  $s$  at large energies!

If there is a small helicity-flip as discussed above, (17) would be altered to

$$A_{NN}^{\bar{p}p} \sim \frac{m_N^2}{s} \quad \text{at fixed } \theta. \quad (18)$$

Of course none of these theoretical predictions can accommodate the rich structure seen in  $A_{NN}^{pp}$  at lower energies (Fig. 3).

Consider now  $A_{LL}$  and  $A_{SS}$ . For  $pp$  at  $\theta = 90^\circ$  there are strong consequences<sup>4</sup> of having zero helicity-flip amplitudes. Namely

$$\begin{aligned} A_{LL}^{pp}(\pi/2) &= 2A_{NN}^{pp}(\pi/2) - 1 \\ A_{SS}^{pp}(\pi/2) &= -A_{NN}^{pp}(\pi/2). \end{aligned} \quad (19)$$

These should be tested!

At other angles, and for  $\bar{p}p$ , we expect moderately large asymmetries because the underlying parton asymmetries are large. Figure 4 shows  $\hat{A}_{LL}$  for various partonic subprocesses.<sup>15</sup> The detailed hadronic predictions depend upon convoluting the partonic asymmetry with the hadron wave-function. It is not clear how much the new EMC results will affect earlier calculations.<sup>16</sup>

### Conclusions

- i) Small  $t$ : We have argued that the natural framework for this region is still the language of Regge poles. Many aspects of the theory, which have never before been tested, can now be scrutinized using polarization type measurements of both the single spin and correlation type. Interesting questions about pomeron and odderon helicity-flip can be answered.
- ii) Large  $p_T$ : Perturbative QCD makes very strong predictions in this region. It will be important to test:

- a) The size and energy variation of the analyzing power,
- b) The relative sign of  $pp$  and  $\bar{p}p$  analyzing powers,
- c) The energy variation of the correlation parameters,
- d) The relative size of the correlation parameters in  $pp$  and  $\bar{p}p$ .

In all, it appears that an extremely rich and fruitful program of experiments awaits us in the very near future.

#### Acknowledgements

I am grateful to the Institute of Theoretical Physics, Santa Barbara, for its warm hospitality, and to the United Kingdom SERC for financial aid. I acknowledge useful discussions with Drs. M. Anselmino and J. Soffer. This research was supported in part by the National Science Foundation under Grant No. PHY82-17853, supplemented by funds from the National Aeronautics and Space Administration, at the University of California at Santa Barbara.

#### References

1. L.N. Lipatov, *Soviet Phys. JETP* **63**, 904 (1986).
2. See *e.g.*, P.D.P. Collins: *An introduction to Regge poles and high energy physics* (Cambridge University Press, 1977).
3. G.C. Fox and E. Leader, *Phys. Rev. Lett.* **18**, 628, 765 (1967).
4. For an overview, see C. Bourrely, E. Leader and J. Soffer, *Phys. Rep.* **59**, 95 (1980).
5. For literature on the odderon, see P. Gauron, E. Leader and B. Nicolescu, *Phys. Rev. Lett.* **54**, 2656 (1985); **55**, 639 (1985).
6. E. Leader, *Phys. Rev. Lett.* **59**, 1525 (1987).
7. E. Leader and R.C. Slansky, *Phys. Rev.* **148**, 1491 (1966); erratum **156**, 1742 (1967); E.L. Berger, A.C. Irving and C. Sorenson, *Phys. Rev.* **D17**, 2971 (1978); E.L. Berger, J.T. Donohue and G. Mennessier, *Phys. Rev.* **D19**, 2595 (1979).

8. G. Lepage and S. Brodsky, *Phys. Rev. D***22**, 2157 (1980).
9. This is extrapolated from the results of N.H. Buttimore, E. Gotsman and E. Leader, *Phys. Rev. D***18**, 694 (1978).
10. M. Anselmino, P. Kroll and B. Pire, *Z. Phys. C***36**, 89 (1987).
11. J.P. Ralston and B. Pire, *Phys. Rev. Lett.* **57**, 2330 (1986).
12. B. Pire and J.P. Ralston, *Phys. Lett.* **117B**, 233 (1982).
13. G.L. Farrar, S. Gottlieb, D. Sivers and G.H. Thomas, *Phys. Rev. D***20**, 202 (1979).
14. M. Anselmino and E. Leader, *Phys. Lett.* **B184**, 261 (1987).
15. N.S. Craigie, K. Hidaka, M. Jacob and F.M. Renard, *Phys. Rep.* **99**, 70 (1983).
16. J. Ashman, *et al.*, *Phys. Lett.* **B206**, 364 (1988).

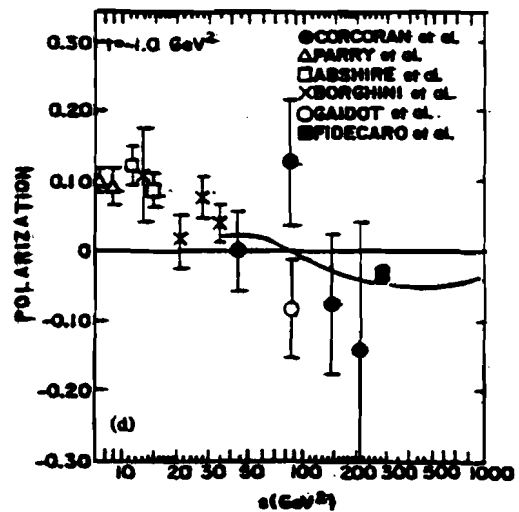
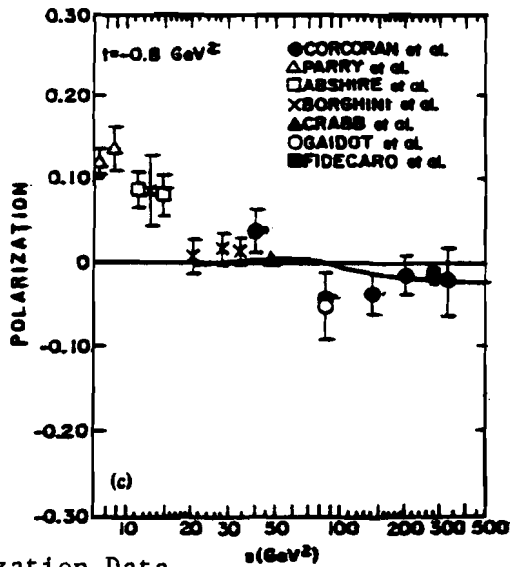
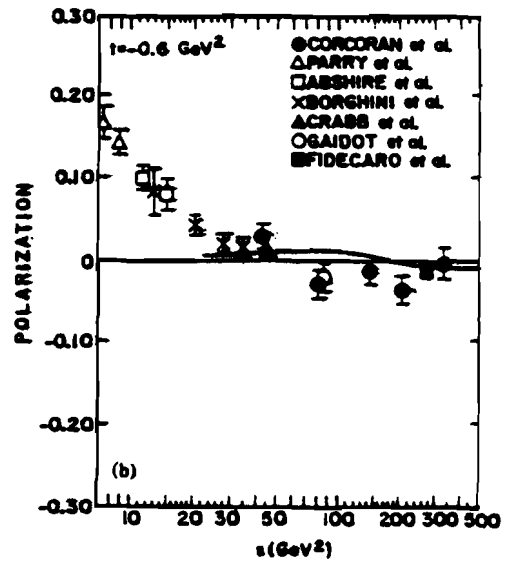
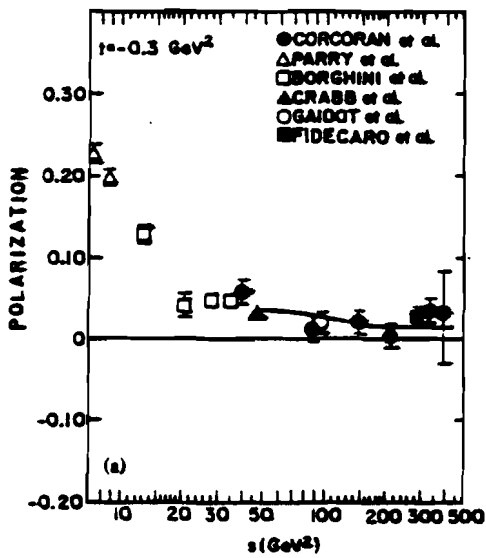
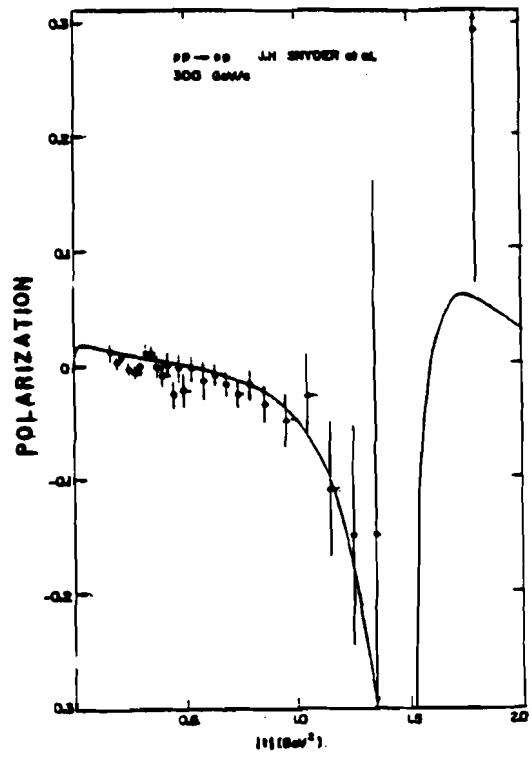
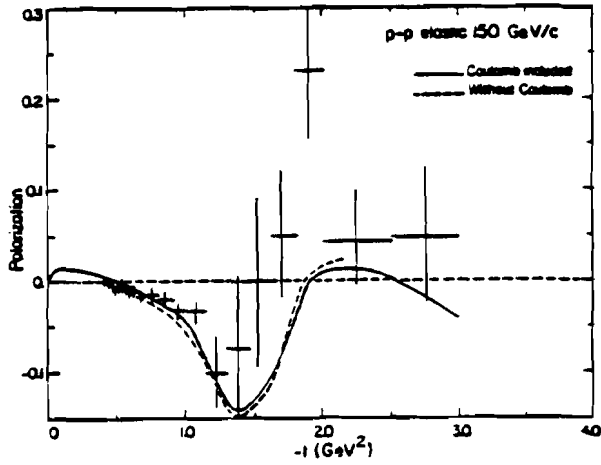


Figure 1 Polarization Data

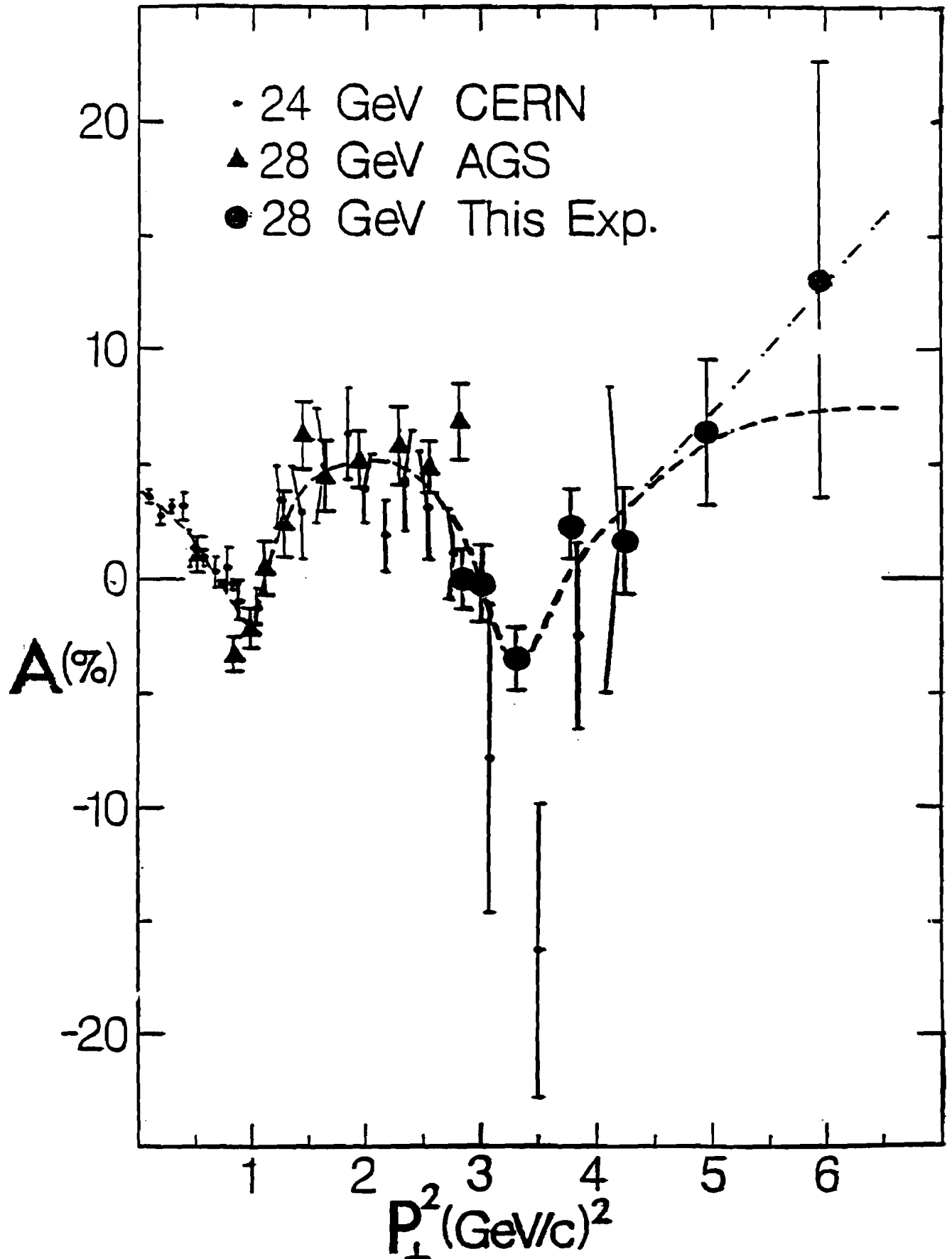
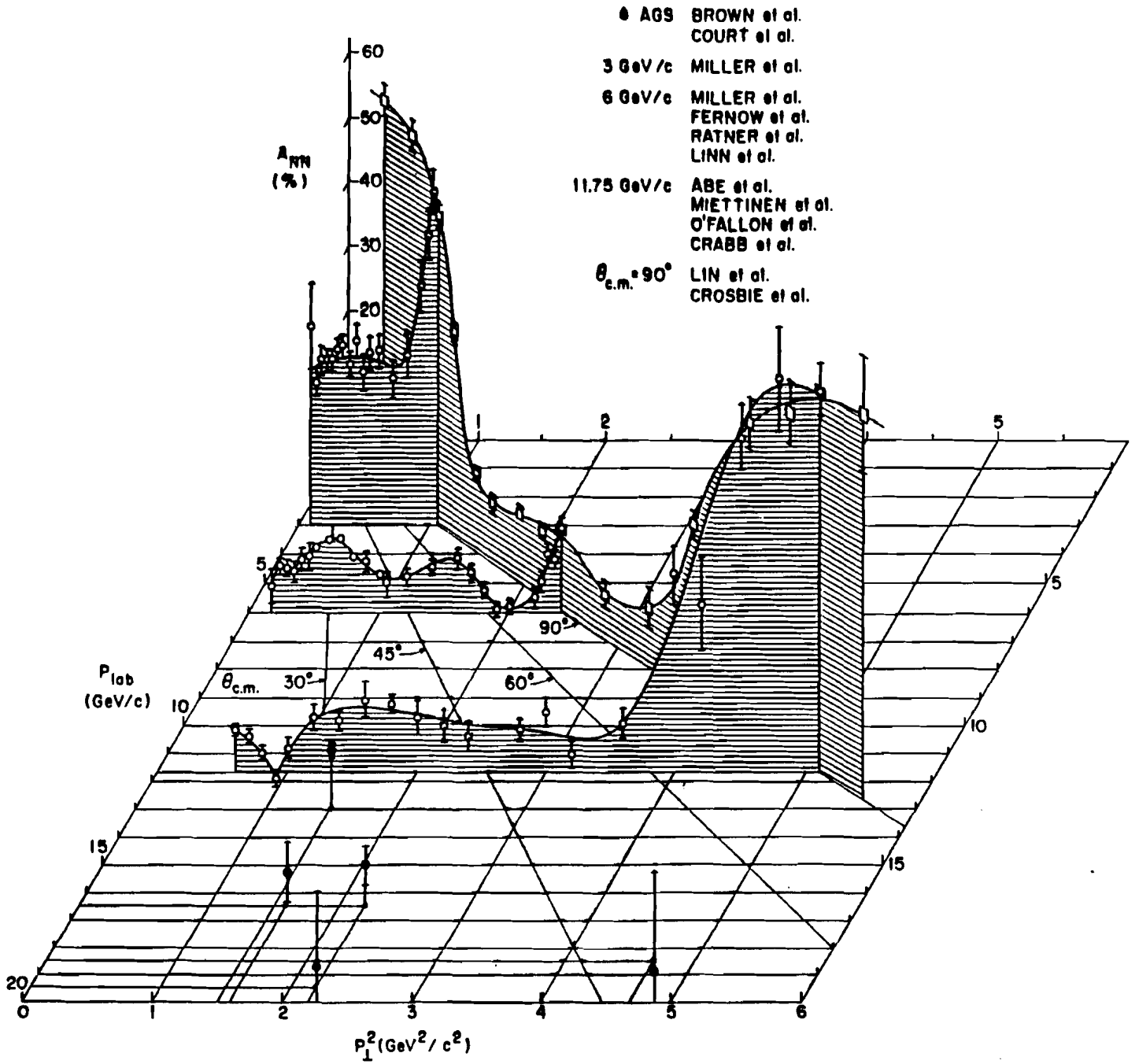


Figure 2 A parameter in the pp elastic scattering.

Figure 3 Structure in  $A_{NN}$ .



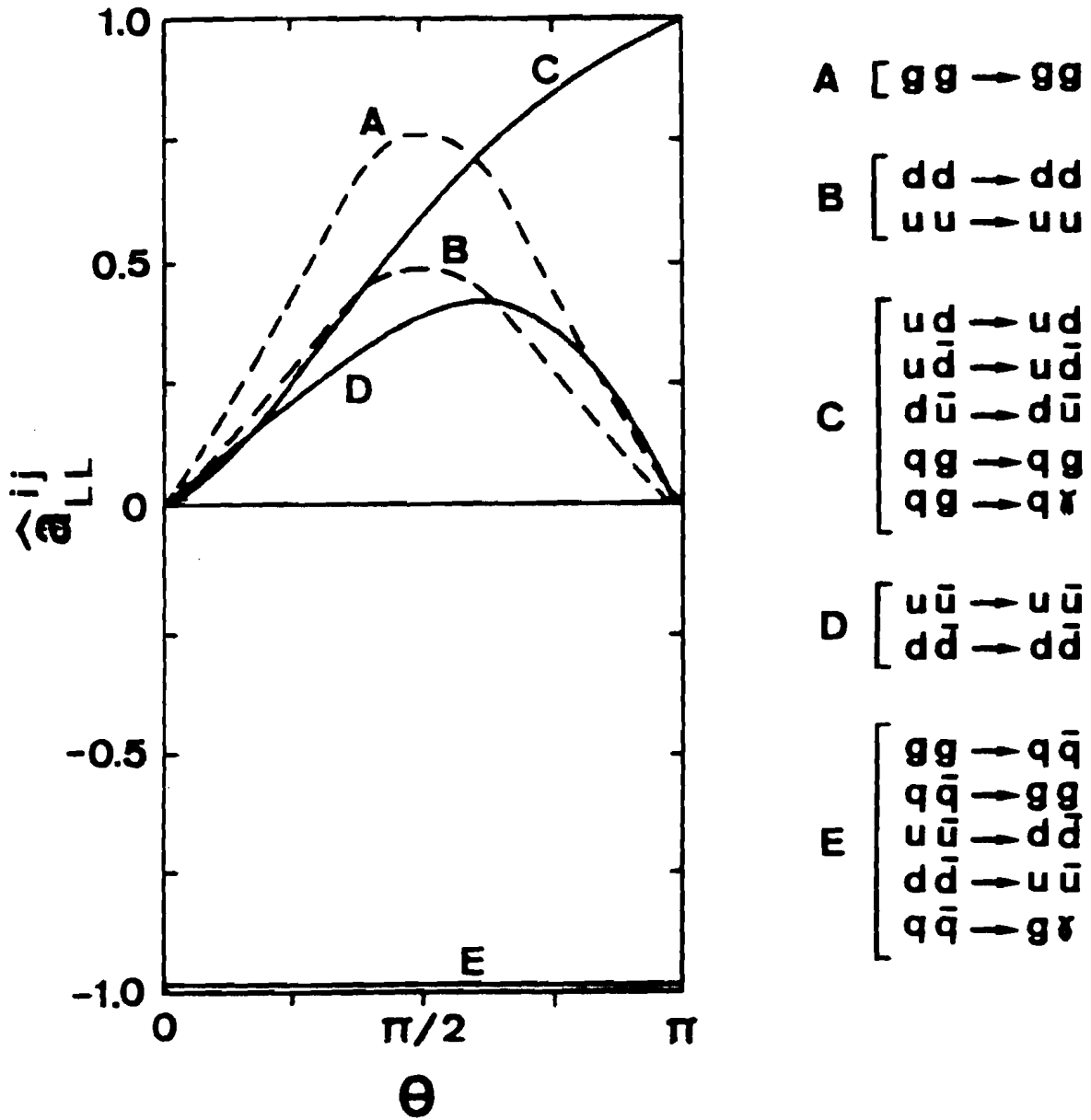


Figure 4:  $\hat{A}_{LL}$  for various partonic processes (from Ref. 15).

